

# Constructing stabilized brane world models in five-dimensional Brans-Dicke theory

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## Abstract

We consider brane world models, which can be constructed in the five-dimensional Brans-Dicke theory with bulk scalar field potentials suggested by the supergravity theory. For different choices of the potentials and parameters we get: (i) an unstabilized model with the Randall-Sundrum solution for the metric and constant solution for the scalar field; (ii) models with flat background and tension-full branes; (iii) stabilized brane world models, one of which reproduces the Randall-Sundrum solution for the metric and gives an exponential solution for the scalar field. We also discuss the relationship between solutions in different frames – with non-minimal and minimal coupling of the scalar field.

## 1 Introduction

Brane world models and their phenomenology have been widely discussed in the last years [1, 2, 3, 4, 5, 6]. One of the most interesting brane world models is the Randall-Sundrum model with two branes, – the RS1 model [7]. This model solves the hierarchy problem due to the warp factor in the metric and predicts an interesting new physics in the TeV range of energies. A flaw of the RS1 model is the presence of a massless scalar mode, – the radion, which describes fluctuations of the branes with respect to each other. As a consequence, one gets a scalar-tensor theory of gravity on the branes, the scalar component being described by the radion. The radion coupling to matter on the negative tension brane contradicts the existing restrictions on the scalar component of the gravitational interaction (see, for example, review [8]), and in order the model be phenomenologically acceptable the radion must acquire a mass. The latter is equivalent to the stabilization of the brane separation distance, i.e. it must be defined by the model parameters. The models, where the interbrane distance is fixed in this way, are called stabilized models, unlike the unstabilized models, where the interbrane distance can be arbitrary.

Such a stabilization of the interbrane distance can be achieved, for example, by introducing a five-dimensional scalar field with brane potentials [9]. A disadvantage of this approach is that the backreaction of the scalar field on the background metric is not taken into account. This problem is solved in the well-known model proposed in [10]. However the background metric of this solution is rather complicated and differs significantly from the Randall-Sundrum solution. An interesting problem is, whether it is possible to find a stabilized model, where the Randall-Sundrum form of the metric is retained despite the interaction with the scalar field.

A solution to this problem based on a non-minimal coupling of the scalar field to gravity was put forward in [11], where the form of the metric was found to be the Randall-Sundrum one, whereas the solution for the scalar field again turned out to be rather complicated.

One of the standard forms of the non-minimal coupling of the scalar field to gravity is the linear coupling to scalar curvature used in the Brans-Dicke theory of gravity. Brans-Dicke theory in the brane world context was already discussed in the literature, see, for example, [12, 13, 14].

In the present paper we use this type of coupling to construct a stabilized model with the Randall-Sundrum solution for the metric, the solution for the scalar field being also a simple exponential function. Our paper is organized as follows. First, we present a method for constructing different background solutions in five-dimensional Brans-Dicke theory by considering bulk and brane scalar field potentials of a special form and examine their correspondence with the solutions in the Einstein frame. In particular, we show that the solutions with these special scalar field potentials in the Jordan frame correspond to the solutions in the Einstein frame with potentials suggested by the supergravity theory. Then we discuss models which can be obtained with different choices of the potentials and parameters. And finally, we discuss the obtained results.

## 2 Setup

Let us denote the coordinates in five-dimensional space-time  $E = M_4 \times S^1/Z_2$  by  $\{x^M\} \equiv \{x^\mu, y\}$ ,  $M = 0, 1, 2, 3, 4$ ,  $\mu = 0, 1, 2, 3$ , the coordinate  $x^4 \equiv y$ ,  $-L \leq y \leq L$  parameterizing the fifth dimension. It forms the orbifold, which is realized as the circle of the circumference  $2L$  with the points  $y$  and  $-y$  identified. Correspondingly, the metric  $g_{MN}$  and the scalar field  $\phi$  satisfy the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x, -y) &= g_{\mu\nu}(x, y), & g_{\mu 4}(x, -y) &= -g_{\mu 4}(x, y), \\ g_{44}(x, -y) &= g_{44}(x, y), & \phi(x, -y) &= \phi(x, y). \end{aligned} \quad (1)$$

The branes are located at the fixed points of the orbifold,  $y = 0$  and  $y = L$ .

The action of the brane world models can be written as

$$\begin{aligned} S &= \int d^4x \int_{-L}^L dy \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right] - \\ &- \int_{y=0} \sqrt{-\tilde{g}} \lambda_1(\phi) d^4x - \int_{y=L} \sqrt{-\tilde{g}} \lambda_2(\phi) d^4x. \end{aligned} \quad (2)$$

Here  $V(\phi)$  is a bulk scalar field potential and  $\lambda_{1,2}(\phi)$  are brane scalar field potentials,  $\tilde{g} = \det \tilde{g}_{\mu\nu}$ ,  $\tilde{g}_{\mu\nu}$  denotes the metric induced on the branes and  $\omega$  is the five-dimensional Brans-Dicke parameter. The formal restriction on this parameter in the five-dimensional Brans-Dicke theory is  $\omega > -\frac{4}{3}$  (which can be easily seen, for example, from the formulas of paper [15]), and further restrictions on its value can be obtained only after studying an effective four-dimensional theory for a certain background solution. The only difference from the classical Brans-Dicke theory is the presence of the bulk scalar field potential  $V(\phi)$  and the branes. The signature of the metric  $g_{MN}$  is chosen to be  $(-, +, +, +, +)$ .

The standard ansatz for the metric and the scalar field, which preserves the Poincaré invariance in any four-dimensional subspace  $y = \text{const}$ , looks like

$$ds^2 = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \phi(x, y) = \phi(y), \quad (3)$$

$\eta_{\mu\nu}$  denoting the flat Minkowski metric. If one substitutes this ansatz into the equations corresponding to action (2), one gets a rather complicated system of nonlinear differential equations for functions  $\sigma(y), \phi(y)$ :

$$3\sigma''\phi + \frac{\omega}{\phi}(\phi')^2 + \phi'' - \sigma'\phi' + \frac{\lambda_1}{2}\delta(y) + \frac{\lambda_2}{2}\delta(y-L) = 0, \quad (4)$$

$$6(\sigma')^2\phi - \frac{1}{2}\left(\frac{\omega}{\phi}(\phi')^2 - V\right) + 4\sigma'\phi' = 0, \quad (5)$$

$$\begin{aligned} & \frac{\omega}{\phi}(\phi'' + 4\sigma'\phi') - 4\sigma'' - 10(\sigma')^2 - \frac{\omega}{2\phi^2}(\phi')^2 = \\ & = \frac{1}{2}\frac{dV}{d\phi} + \frac{1}{2}\frac{d\lambda_1}{d\phi}\delta(y) + \frac{1}{2}\frac{d\lambda_2}{d\phi}\delta(y-L), \end{aligned} \quad (6)$$

where prime denotes the derivative with respect to extra dimension coordinate  $y$ .

We will consider a special class of bulk potentials, namely

$$V(\phi) = -(3\omega + 4) \left[ (4\omega + 5) \phi F^2 + 2\phi^2 F \frac{dF}{d\phi} - 3\phi^3 \left( \frac{dF}{d\phi} \right)^2 \right], \quad (7)$$

where  $F \equiv F(\phi)$  is a function. This structure of the potentials is similar to the one introduced in [16] for the case of the minimal coupling of the scalar field to gravity. Although the potential in its general form (7) looks rather complicated, the resulting potentials for different choices of the function  $F$ , as we will see below, look quite natural.

One can check that in this case any solution of the equations

$$\phi' = \phi F - 3\phi^2 \frac{dF}{d\phi}, \quad (8)$$

$$\sigma' = (\omega + 1) F + \phi \frac{dF}{d\phi} \quad (9)$$

also satisfies (4), (5) and (6) in the interval  $[0, L]$ , provided that the following boundary conditions on the branes are satisfied

$$(3\omega + 4) F|_{y=+0} = -\frac{\lambda_1}{4\phi}, \quad (10)$$

$$(3\omega + 4) F|_{y=L-0} = \frac{\lambda_2}{4\phi}, \quad (11)$$

$$(3\omega + 4) \frac{dF}{d\phi}|_{y=+0} = -\frac{1}{4} \frac{d(\lambda_1/\phi)}{d\phi}, \quad (12)$$

$$(3\omega + 4) \frac{dF}{d\phi}|_{y=L-0} = \frac{1}{4} \frac{d(\lambda_2/\phi)}{d\phi}. \quad (13)$$

It is necessary to note that the symmetry conditions (1) were used to obtain these relations.

Thus we get first order differential equations instead of the initial second order differential equations. The situation is analogous to that in papers [10, 16], where the bulk and the brane

potentials for the scalar field minimally coupled to five-dimensional gravity are chosen in an appropriate way.

It is a common knowledge that action (2), called the action in the Jordan frame, can be brought by a conformal transformation to an action with the scalar field minimally coupled to gravity, which is called the action in the Einstein frame. Indeed, if in action (2) we put  $\phi = 2M^3 \exp(-\frac{3\rho}{a})$  with  $a = 6M^{3/2}(\omega + 4/3)^{1/2}$  and make a conformal rescaling  $g_{MN} \rightarrow \exp(\frac{2\rho}{a})g_{MN}$  allowing one to pass from the Jordan frame to the Einstein frame, we get a model, describing scalar field  $\rho$  minimally coupled to five-dimensional gravity, with the action

$$\begin{aligned} S = & \int d^4x \int_{-L}^L dy \sqrt{-g} \left( 2M^3 R - \frac{1}{2} g^{MN} \partial_M \rho \partial_N \rho - \bar{V}(\rho) \right) - \\ & - \int_{y=0} \sqrt{-\tilde{g}} \bar{\lambda}_1(\rho) d^4x - \int_{y=L} \sqrt{-\tilde{g}} \bar{\lambda}_2(\rho) d^4x, \end{aligned} \quad (14)$$

where

$$\bar{V}(\rho) = e^{\frac{5\rho}{a}} V(2M^3 e^{-\frac{3\rho}{a}}), \quad \bar{\lambda}_i(\rho) = e^{\frac{4\rho}{a}} \lambda_i(2M^3 e^{-\frac{3\rho}{a}}), \quad i = 1, 2.$$

Thus, if we have a solution in the Jordan frame

$$ds^2 = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \phi(x, y) = \phi(y),$$

generated by potential (7), it can be transformed to a solution of the corresponding theory (14) in the Einstein frame

$$ds^2 = e^{2(\sigma(y) - \frac{\rho(y)}{a})} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-\frac{2\rho}{a}} dy^2, \quad \rho(y) = -\frac{a}{3} \ln \left( \frac{\phi(y)}{2M^3} \right).$$

To bring the metric of this solution to the standard form (3), we have to pass to a new coordinate  $z$  of the extra dimension according to  $dz = \exp(-\frac{\rho(y)}{a}) dy$ . Thus, we get a solution in the Einstein frame

$$ds^2 = e^{2(\sigma(z) - \frac{\rho(z)}{a})} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \quad \rho(z) = \rho(y(z)),$$

in theory (14) with potentials

$$\bar{V}(\rho(z)) = \frac{1}{8} \left( \frac{dW}{d\rho} \right)^2 - \frac{1}{24M^3} W^2, \quad (15)$$

$$W(\rho(z)) = -\frac{2a^2}{3} e^{\frac{\rho(z)}{a}} F(2M^3 e^{-\frac{3\rho}{a}}), \quad (16)$$

$$\bar{\lambda}_i(\rho(z)) = \pm W(\rho(z))|_{z=z_i}, \quad i = 1, 2, \quad (17)$$

which have exactly the same form as the potentials of the model discussed in [10, 16], where their form was suggested by the supergravity theory. Thus, the method of finding solutions in the Brans-Dicke theory is equivalent to the one discussed in these papers, which is quite clear, because in both cases the second order differential equations are reduced to the first order ones. Nevertheless, if we study a Brans-Dicke theory, it is more convenient to look for solutions in the Jordan frame than to transform them from the Einstein frame.

Another point is that solutions in one frame (Jordan or Einstein) may be elegant and interesting in view of the hierarchy problem, whereas in the other frame it is not the case. For

example, if in the Jordan frame we take polynomial functions  $F$ , in the Einstein frame we get solutions for theories with complicated potentials being a sum of several exponential functions. Of course, the converse is also true. For this reason it is also convenient to have a method of constructing brane world solutions directly in the Jordan frame.

One more important point is that though solutions in different frames are equivalent, the corresponding theories in different frame are not equivalent, because if in the Brans-Dicke theory we had the minimal coupling of gravity to matter on the branes, then after the conformal transformation the scalar field would enter the term describing the interaction with matter on the branes (the so-called conformal ambiguity [17]). At the same time, since the model described in [10] is stable against fluctuations of gravitational and scalar fields, i.e. it is tachyon- and ghost-free (see [18]), the model described by (2) and (7) should possess this property too, although the bulk potentials for certain choices of  $F$  are unbounded from below.

### 3 Specific examples

#### 3.1 The Randall-Sundrum solution

Let us choose the function  $F(\phi)$  in the following form

$$F = B\phi^{\frac{1}{3}}, \quad (18)$$

where  $B$  is a constant. It follows from (8) that  $\phi = \text{const}$  in this case. Bulk and brane potentials are

$$V(\phi) = -\frac{4}{3}B^2\phi^{\frac{5}{3}}(3\omega + 4)^2, \quad (19)$$

$$\lambda_{1,2} = \mp 4B(3\omega + 4)\phi^{\frac{4}{3}}, \quad (20)$$

and solution for the warp factor is

$$\sigma' = \frac{B}{3}(3\omega + 4)\phi^{\frac{1}{3}} \quad (21)$$

in the interval  $[0, L]$ . Making redefinition

$$\frac{\sqrt{2}}{3}B(3\omega + 4)\phi^{\frac{1}{3}} = -k, \quad (22)$$

$$\phi = 2M^3, \quad (23)$$

we get

$$V(\phi) = -12k^2M^3, \quad (24)$$

$$\lambda_{1,2} = \pm 12kM^3, \quad (25)$$

which formally coincide with the original RS1 solution [7]. Nevertheless the linearized theory in this background differs from that in the RS1 model, because the fluctuations of the scalar field add an extra degree of freedom. In this case the model cannot be stabilized (i.e. the size of the extra dimension cannot be fixed) by adding positively defined potentials on the branes, since the solution for the scalar field does not depend on  $y$  (this point is discussed in detail in

subsection 3.3). Thus, this example is not interesting from the physical point of view, but it shows how the general method for constructing solutions works.

It is not difficult to see that action (2) with potentials of the form (19) and (20) can be brought by a conformal transformation of the metric and coordinate transformations, described in Section 2, to the action of the unstabilized RS1 model with minimally coupled scalar field. The only significant difference is that if in the Brans-Dicke theory we had the minimal coupling of gravity to matter on the branes, then after the transformation the scalar field would enter the term describing the interaction with matter on the branes.

### 3.2 "Consistent" ADD scenario

Let us consider the case, where  $\omega = 0$ , i.e. the kinetic term for the Brans-Dicke scalar field is absent. Let us also suppose that  $F = B/\phi$ . It is not difficult to check that

$$V(\phi) \equiv 0, \quad (26)$$

$$\sigma' = 0 \quad (27)$$

in this case. Equation (27) means that we have the flat five-dimensional background metric. Solution for  $\phi$ , following from (8), has the form

$$\phi = 4By + D \quad (28)$$

in the interval  $[0, L]$ , where  $D$  is a constant. From the boundary conditions one gets

$$\lambda_{1,2} = \mp \lambda, \quad (29)$$

where  $\lambda$  is a constant defining the brane tensions. Finally we get

$$\phi = \frac{\lambda}{4}|y| + D. \quad (30)$$

Thus, we get the model with the flat five-dimensional background metric and tension-full branes, which was discussed in detail in paper [19]. This model can be easily stabilized by the same method as the one, which will be discussed in the next subsection.

### 3.3 Stabilized brane world with the Randall-Sundrum solution for the metric

Now let us consider the case  $F = \text{const}$ , i.e. it does not depend on the field  $\phi$ . Bulk and brane potentials, corresponding to such a choice of  $F(\phi)$ , can be chosen to be (see (7) and (10)–(13))

$$V(\phi) = \Lambda\phi, \quad (31)$$

$$\lambda_{1,2} = \pm \lambda\phi. \quad (32)$$

Let us suppose that  $\lambda > 0$ . It is not difficult to check that from (8)–(13) follows

$$\sigma = -k|y|, \quad (33)$$

$$\phi = Ce^{-u|y|} \quad (34)$$

with

$$u = \sqrt{\frac{-\Lambda}{(3\omega + 4)(4\omega + 5)}}, \quad (35)$$

$$k = (\omega + 1)u, \quad (36)$$

and the fine-tuning relation

$$\lambda = 4\sqrt{-\Lambda}\sqrt{\frac{3\omega + 4}{4\omega + 5}}. \quad (37)$$

Constant  $C$  is not defined by the equations. One can see that in the limit  $\omega \rightarrow \infty$  we arrive at the standard Randall-Sundrum solution.

Now let us discuss stabilization mechanism which can be utilized in the case under consideration. We will follow the way proposed in [10] and add stabilizing quadratic potentials on the branes, namely

$$\Delta\lambda_{1,2} = \gamma_{1,2}(\phi - v_{1,2})^2, \quad \gamma_{1,2} > 0. \quad (38)$$

Such an addition will not affect equations of motion provided

$$\phi|_{y=0} = v_1, \quad \phi|_{y=L} = v_2. \quad (39)$$

Thus, now the constant  $C$  appears to be defined and is equal  $C = v_1$ , whereas the size of extra dimension is now defined by the relation

$$L = \frac{1}{u} \ln \left( \frac{v_1}{v_2} \right), \quad (40)$$

which is the same as in the model of paper [10]. Obviously, this stabilization mechanism works only for solutions with the scalar field depending on the extra coordinate, in particular, for the solution in subsection 3.2, and does not work for the solution in subsection 3.1. We would like to emphasize that this mechanism differs somewhat from the Goldberger-Wise mechanism [9], because it takes into account the backreaction of the scalar field on the metric, and the size of the extra dimension is fixed by the boundary values of the former.

Now let us find the relationship between the four-dimensional Planck mass and the parameters of the theory. We assume that the brane at  $y = L$  is "our" brane. To this end one should choose  $\sigma = -k|y| + kL$  to make four-dimensional coordinates  $\{x^\mu\}$  Galilean on this brane (this problem was discussed in detail in [20]). Naive considerations suggest (see, for example, [19]) that the wave function of the massless four-dimensional tensor graviton has the same form as that in the unstabilized Randall-Sundrum model, namely  $h_{\mu\nu}^0(x, y) = e^{2\sigma}\alpha_{\mu\nu}(x)$ . Moreover, the form of the residual gauge transformation, which are left after imposing the gauge on the fluctuations of metric (see [18, 20]), also suggests the same form of the wave function. Thus, substituting the following ansatz

$$\begin{aligned} g_{\mu\nu}(x, y) &= e^{2\sigma}g_{\mu\nu}^{(4)}(x), \quad \phi(x, y) = \phi(y), \\ g_{44}(x, y) &= 1, \quad g_{\mu 4}(x, y) = 0 \end{aligned} \quad (41)$$

into action (2), we get

$$S = \int_{-L}^L \phi e^{2\sigma} dy \int R_{(4)} \sqrt{-g_{(4)}} d^4x \quad (42)$$

and

$$2M_{Pl}^2 = \int_{-L}^L \phi e^{2\sigma} dy = \frac{2v_1}{2k+u} (e^{2kL} - e^{-uL}). \quad (43)$$

If  $uL < 1$  we get a formula analogous to that in the unstabilized Randall-Sundrum model (see [6, 20]). To solve the hierarchy problem one needs  $kL$  to be of the order  $kL \sim 30$ . If one chooses relatively large  $\omega$  (for example,  $\omega \geq 30$ ), then  $k$  would go to the value corresponding to the unstabilized Randall-Sundrum model, namely

$$k \approx \sqrt{\frac{-\Lambda}{12}}, \quad (44)$$

(compare with (23) and (24)) whereas  $uL$  could be made less than unity ( $uL < 1$ ), since  $u = k/(\omega + 1)$ . Under this assumption the parameter  $v_1$  can be chosen to be of the same order as  $v_2$ . In this case the parameters of the model, made dimensionless with the help of a fundamental scale in the  $TeV$  range, do not contain a hierarchical difference. The situation turns out to be completely analogous to that in the model proposed in [10]. At the same time, solution for the warp factor in stabilized brane world model found in [10] is quite complicated, which impedes the analysis of the equations of motion for linearized gravity (approximate solutions can be found only under certain assumptions and simplifications, see [18]). As for our case, one can think that though the general structure of action (2) is more complicated than that of the action used in [10], the simplicity of solutions (33) and (34) could result in simpler equations of motion for linearized gravity.

Another advantage of the solution presented above is that one can use all the results, obtained for the case of the universal extra dimensions in the Randall-Sundrum model (i.e. if one allows additional fields to propagate in the bulk, see, for example, [21] and references therein), in our case too. This happens because of the equivalence of solutions for the warp factors in both models. Of course, it is true in the case of the standard coupling of five-dimensional gravity to matter in the bulk. But since the size of extra dimension in our model appears to be stabilized, one can think that this would allow us to avoid possible problems caused by the radion field, which are inherent in the unstabilized Randall-Sundrum model.

Quite an interesting situation arises, if one chooses  $\omega = -1$ . Although the Brans-Dicke parameter is negative, the model is stable, since  $\omega > -\frac{4}{3}$ . Formulas (33)–(37) with  $\omega = -1$  take the form

$$\sigma = 0, \quad (k = 0), \quad (45)$$

which means that the five-dimensional background metric is flat,

$$\phi = C e^{-u|y|}, \quad (46)$$

$$u = \sqrt{-\Lambda}, \quad (47)$$

$$\lambda = 4\sqrt{-\Lambda}. \quad (48)$$

Thus we get a model, which is similar, to some extent, to that discussed in subsection 3.2. In order to have the hierarchy problem solved (in the way proposed in [19]) in case of  $TeV$



range of fundamental five-dimensional physics, one should choose  $uL \sim 30$ , as in the Randall-Sundrum model, and  $C \sim e^{uL}$ . The flaw of the case  $\omega = -1$  is that there appears a new hierarchy between  $v_1$  and  $v_2$ . Nevertheless, this choice of parameters can be interesting from the pedagogical point of view since it demonstrates another scenario with flat background and tension-full branes (and nonempty bulk).

The solutions of this section can be easily related to solutions in the Einstein frame. For these solutions  $F = \text{const}$ , and  $\bar{V}(\rho(z))$  (see (15) and (16)) is the Liouville potential. Such "dilaton" brane worlds were widely discussed in the literature, see, for example [22, 23]. As we have shown, our solutions can be brought to the form of the general solutions of paper [22] by a conformal rescaling of the metric and an appropriate transformation of the extra dimension coordinate, the latter being necessary for retaining the same ansatz for the metric (of the form (3)).

Our results also give a simple solution to the problem of finding a stabilized brane world model with the Randall-Sundrum form of the metric, which was discussed in [11]. In this paper a similar solution in a theory with the scalar field non-minimally coupled to gravity, which preserves the Randall-Sundrum form of the metric, was found. But this solution and ours cannot be transformed to each other by a redefinition of fields, rescaling of metric and coordinate transformations.

We would like to note once again that the theories obtained by a rescaling of the metric are not equivalent, if one considers the standard coupling of gravity to matter on the branes, as we have already mentioned in the end of Section 2. Since in stabilized brane world models the fluctuations of the scalar field describe also the radion (see [18]), this ambiguity modifies, in particular, the radion coupling to matter on the branes. Because we do not know, which frame is the "real" one, there are no strong objections against choosing the Jordan one. In this connection, an interesting problem is to compare the physical consequences of the models in different frames, which can be transformed one into another in the absence of matter on the branes.

### 3.4 Power law solutions

Let us consider the case

$$F = B\phi^n, \quad (49)$$

where  $n \neq \frac{1}{3}$  and  $n \neq 0$  (these cases were discussed above). The corresponding bulk potential has the form

$$V(\phi) = -(3\omega + 4)B^2\phi^{2n+1} [4\omega + 5 + 2n - 3n^2], \quad (50)$$

whereas the fine-tuned brane potentials can be easily obtained from (10)–(13). Solutions for the scalar field and the warp factor, following from (8)–(13), are

$$\sigma = \frac{\omega + 1 + n}{(3n - 1)n} \ln [n(3n - 1)B|y| + C] + C_1, \quad (51)$$

$$\phi = [n(3n - 1)B|y| + C]^{-\frac{1}{n}}, \quad (52)$$

where constants  $C$  and  $C_1$  are not defined by the equations. We will consider only such values of parameters  $n$ ,  $B$  and  $C$  that the expression  $[n(3n - 1)B|y| + C]$  is positive for any value of  $y$ .

Let us suppose that that "our" brane is that at the point  $y = 0$  (not that at the point  $y = L$ , as in the previous example). In this case we should take

$$C_1 = -\frac{\omega + 1 + n}{(3n - 1)n} \ln(C) \quad (53)$$

to make four-dimensional coordinates  $\{x^\mu\}$  be Galilean on this brane (i.e.  $\sigma(y = 0) = 0$ ). The constant  $C$  is defined by the stabilizing potential on the brane at  $y = 0$ , whereas the size of the extra dimension is defined by the stabilizing potential on the brane at  $y = L$ .

After some algebra we can easily get from (42) the value of effective four-dimensional Planck mass on the brane at  $y = 0$

$$\begin{aligned} M_{Pl}^2 &= \int_0^L \phi e^{2\sigma} dy = \\ &= \frac{C^{\frac{n-1}{n}}}{B(2\omega + 3 + 3n^2 - 2n)} \left[ \left( \frac{n(3n-1)BL}{C} + 1 \right)^{\frac{2\omega+3+3n^2-2n}{n(3n-1)}} - 1 \right]. \end{aligned} \quad (54)$$

We will show why such solutions can be interesting from the point of view of hierarchy problem by utilizing the choice<sup>1</sup>

$$n = \frac{3}{2}. \quad (55)$$

Bulk and brane potentials, corresponding to such a choice of  $n$ , have the form

$$V(\phi) = -(3\omega + 4) \frac{16\omega + 5}{4} B^2 \phi^4, \quad (56)$$

$$\lambda_{1,2} = \mp 4B(3\omega + 4) \phi^{\frac{5}{2}}. \quad (57)$$

Solutions for the scalar field and the warp factor, following from (51), (52) and (53), are

$$\sigma = \frac{4\omega + 10}{21} [\ln(21B|y| + D) - \ln(D)], \quad (58)$$

$$\phi = \left( \frac{4}{21B|y| + D} \right)^{\frac{2}{3}}, \quad (59)$$

where  $D$  is a constant which will be defined below.

To stabilize the size of the extra dimension, we add potentials of the form (38) on the branes:

$$\Delta\lambda_{1,2} = \gamma_{1,2} (\phi - v_{1,2})^2. \quad (60)$$

As in the case discussed in the previous subsection, we get

$$\phi|_{y=0} = v_1, \quad \phi|_{y=L} = v_2. \quad (61)$$

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<sup>1</sup>The authors are grateful to K. Farakos and P. Pasipoularides for suggesting to examine this case and the corresponding background solution, which resulted in this subsection

The constant  $D$  appears to be defined by

$$D = 4 (v_1)^{-3/2}, \quad (62)$$

and we get

$$\phi = v_1 \left( \frac{1}{\frac{21(v_1)^{3/2} B|y|}{4} + 1} \right)^{\frac{2}{3}}. \quad (63)$$

It follows from (61) and (63) that the size of extra dimension is defined by the relation

$$L = \frac{4}{21B} \left[ \frac{(v_1)^{3/2} - (v_2)^{3/2}}{(v_1 v_2)^{3/2}} \right]. \quad (64)$$

The warp factor has the form

$$e^\sigma = \left( \frac{21 (v_1)^{3/2} B|y|}{4} + 1 \right)^{\frac{4\omega+10}{21}}, \quad (65)$$

and it is not difficult to calculate the value of the four-dimensional Planck mass on the brane at  $y = 0$ . Using (54) we can easily get

$$M_{Pl}^2 = \int_0^L \phi e^{2\sigma} dy = \frac{4}{B (v_1)^{1/2} (8\omega + 27)} \left[ \left( \frac{v_1}{v_2} \right)^{\frac{8\omega+27}{14}} - 1 \right]. \quad (66)$$

Let us suppose that all the parameters of the model, made dimensionless with the help of a fundamental scale in the  $TeV$  range, do not contain a hierarchical difference. For example, one can choose  $B \approx 1TeV^{-7/2}$ ,  $v_1 \approx 1TeV^3$  and  $\frac{v_1}{v_2} \approx 3.4$ . It follows from this assumption that  $B (v_1)^{3/2} L \approx 1$ , which means that  $L \approx 1TeV^{-1}$ . If one chooses  $\omega = 110$ , then the four-dimensional Planck mass on the brane at  $y = 0$  appears to be of the order of  $M_{Pl} \sim 10^{19} GeV$ . Thus, although the largest dimensionless parameter in the model is  $\omega = 110$ , we get a difference in 16 orders of magnitude between four- and five-dimensional energy scales. We also see, that in this case the hierarchy problem is solved because of the large power in (66), contrary to the case discussed in the previous subsection, where the hierarchy problem is solved due to the exponential factor (as in the Randall-Sundrum model). Of course, if we take larger value of  $\frac{v_1}{v_2}$ , the value of  $\omega$  can be much smaller.

## 4 Conclusion and final remarks

In this paper we considered five-dimensional Brans-Dicke theory as a basis for building different solutions corresponding to brane world models. It appeared that for particular choices of the potentials and certain values of parameters the theory reproduces some known background solutions. We also presented new solutions for stabilized brane worlds, one of which has a relationship with a known solution in another frame. We hope that appropriate choice of the function  $F(\phi)$  can lead to other interesting solutions, which are not evident at the first glance.

A reasonable question arises – what happens to the mass of the radion and its coupling to matter on the brane, especially in the stabilized cases discussed in subsections 3.3, 3.4? It is clear that the radion mass should be expressed through the model parameters  $\gamma_{1,2}$ ,  $\Lambda$  (or  $B$ , respectively),  $\omega$  and  $v_{1,2}$ . Calculations made in [18] for the stabilized brane world model proposed in [10] suggest that with an appropriate choice of these parameters the radion mass can be made to be in the  $TeV$  range, which can be interesting from the experimental point of view and does not contradict the known data. Nevertheless, an answer to the question posed above can be obtained only after a thorough examination of linearized gravity in the models. This issue calls for further investigation.

We would like to note that all the models found in the present paper have a finite size of extra dimension and are of interest for solving the hierarchy problem. At the same time, it would be interesting to look for "fat brane" solutions in five-dimensional Brans-Dicke theory, analogous to that found in [24], and to examine their properties. This problem also deserves to be studied.

Finally, it is necessary to mention that there are other types of non-minimal coupling of the scalar field to gravity. For example, in recent papers [25, 26, 27] some interesting brane world solutions (both analytic and numerical) in the theory with Ricci-coupled bulk scalar field were found. In this connection it would be interesting to compare the physical consequences of the models with different types of non-minimal coupling and identical solutions for the warp factor – for example, of the model discussed in subsection 3.3 of this paper and of the models discussed in [11, 25, 27], in the case of the standard coupling of gravity to matter on the branes in all the models.

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